Sample SOLUTIONS Guide

 **(a)** To minimize $\frac{1}{2}W^{'}ΣW$

$$subject to r+(μ-r1)^{'}W=0.1$$

**Solution**

Since, there is only one constraint, assume Lagrangian Multiplier be ‘$γ'$.

Lagrange Function is as follows:

|  |  |
| --- | --- |
| $$L\left(W,γ\right)=\frac{1}{2}W^{'}ΣW+ γ(0.1-r-(μ-r1)^{'}W)$$ | (1) |

For maxima or minima,

$$\frac{∂L}{∂W}=0$$

$$⇒W^{'}Σ+γ\left(-\left(μ-r1\right)^{'}\right)=0$$

$$\left\{SinceW has degree 2, differentiation shall cancel the scaling factor 1/2\right\}$$

$$⇒W^{'}Σ=γ\left(\left(μ-r1\right)^{'}\right)$$

Taking transpose on both the sides, we get,

$$(W^{'}Σ)^{'}=γ\left(\left(μ-r1\right)^{'}\right)^{'}$$

$$⇒ Σ^{'}W=γ(μ-r1)$$

But $Σ^{'}=Σ \{property of symmetric matrix\}$

Therefore, $ΣW=γ(μ-r1)$