CQF Module 5 Assignment

January 2017 Cohort

**Marking Scheme: Q1** 30% **Q2-4** 10-10-10% **Credit Curve** 40%

1. You are analyzing a company with the equity of $3 million and its volatility *σE* is 50%. Company’s debt of $5 million notional matures in 1 year, and the risk-free rate is 2%. Build a structural model to report:
   1. Computation of the firm’s assets value of *V*0 and volatility *σV* .

**Note:** set up a system of equations in Excel/Mathematica and use Solver/alike for numerical root-finding.

* 1. Sensitivity of the 1Y PD to the equity volatility input *σE* on the plot.

Compute the sensitivity for both, Merton PD and Black-Cox PD, for which set *K* = $5 million and use analytical results from the Lecture.

Briefly explain the difference in sensitivity *wrt* volatility levels above 60%.

1. Use the definition of hazard rate (intensity) *λ* = *−* = *−* , where survival proba-

*d* log *S*(*x*) *Sr*(*x*)

*dx*

bility S(x) = 1 *−* F(x) for the following tasks:

*S*(*x*)

* 1. Confirm that *λ* is the intensity of the Exponential distribution with

*cdf F* (*x*) = 1 *− e−λx*.

* 1. Find hazard rate (intensity) of a random variable *X* that follows the Pareto distribu- tion with the *cdf*

. Σ

*F* (*x*) = 1 *−*

*β α*

*.*

*x* + *β*

1. For a Poisson process, the inter-arrival time of the *n*th jump is defined as

*τi* = *Tn − Tn−*1. *τi* is an *iid* process that follows the Exponential distribution with

*dF*

*− −λτ*

*F* (*τ* ) = 1 *e* and *f* (*τ* ) =

*dτ*

= *λe−λτ*

* 1. Calculate the expected inter-arrival time (between jumps) E[*τ* ] as the first moment of the Exponential distribution. Integration workings must be provided.
  2. Use the result from step (a) to calculate the expected time of arrival of the *n*th jump

E [*Tn*] given that *Tn* = Σ*n τi*.

*i*=1

1. Consider a risky zero coupon bond *ZI* (*r, t*; *p*) with stochastic short-term interest rate *r* and the risk of default is governed by Poisson process with constant intensity *p*.

*dr*(*t*) = *u*(*r, t*)*dt* + *w*(*r, t*)*dx*

The risky bond can be hedged by a risk-free bond *Z*(*r, t*). Start with the hedging portfolio is Π = *ZI* ∆*Z* and *dr*(*t*) process and derive the PDE for the risky bond that incorporates a constant recovery rate *θ*. Below is the result you obtain,

*−*

*∂ZI* 1 2 *∂*2*ZI*

*∂ZI*

+ *w*

*∂t* 2

*∂r*2 + (*u − λw*) *∂r −* (*r* + (1 *− θ*)*p*)*ZI* = 0

Invoke Feynman-Kac theorem **to write down the expressions** for (a) the fundamental asset pricing formula and (b) forward rates.

**Credit Curve**

Table 1 shows the term structure of credit spreads for two reference entities at the opposite ends of a credit spectrum: Wells Fargo (WFC) is a highly-rated institution, while Clear Channel Communications (CCMO) is highly leveraged according to Fitch Ratings (18 May 2012). Assume CDS premium is paid annually in arrears and in the case of a default, the payment is made at the year end.

|  |  |  |  |
| --- | --- | --- | --- |
| Maturity | WFC | CCMO | *Z*(0; *T* ) |
| 1Y | 50 | 751 | 0.97 |
| 2Y | 77 | 1164 | 0.94 |
| **3Y** | 94 | 1874 | 0.92 |
| **5Y** | 125 | 4156 | 0.86 |
| **7Y** | 133 | 6083 | 0.81 |

Table 1: CDS Market Data

* 1. Bootstrap implied survival probabilities for WFC bank with recovery rate *RR* = 50%. Obtain the term structure of hazard rates for and plot Exponential *pdf f* (*t*) = *λe−λt*. It is your task to plot *pdf* function as appropriate for piecewise constant lambda.

*P* (0*, T* ) = exp .*−* Σ

*T*

*λ* ∆*t*Σ

1

*λ* = *−*

log

*P* (0*, tm*)

*t*

*t*=1

*m* ∆*t*

*P* (0*, t*

*m−*1)

where *P* (0*, T* ) is cumulative PrSurv, while *λm* is hazard rate per period (per annum).

* 1. Bootstrap implied survival probabilities for CCMO corporation. Assume *RR* = 10%. Is there an *anomaly* for this highly-leveraged name? Note any interesting observations.
  2. What is the effect of an increase in recovery rate on implied survival probabilities? Illustrate with the plots of WFC survival probability term structure for different values of *RR*.

**PrSurv bootstrapping from CDS quotes must be coded as a function by you – resubmission of the lecture spreadsheet or non-original code will receive a sizeable deduction in marks.**