CQF Module 4 Examination

Instructions

All questions must be attempted. Books and lecture notes may be referred to. **Any clarification (only) should be e-mailed to riaz.ahmad@fitchlearning.com.**

dW is the usual increment of a Brownian motion.

1. We wish to find the approximate value of a cashflow for a floorlet on the one month LIBOR, when using the Vasicek model. Show that this is given by

$$\max\left(r_f - r - \tfrac{1}{24}\left(\eta - \gamma r\right), 0\right),$$

   where r_f is the floor rate and r the spot rate. **[8 Marks] You <u>MUST</u> start by considering the yield curve power series expression given in the calibration and data analysis lecture. <u>FULL</u> working should be given for the series expansion, or you will lose credit.**

2. Consider the following interest rate data
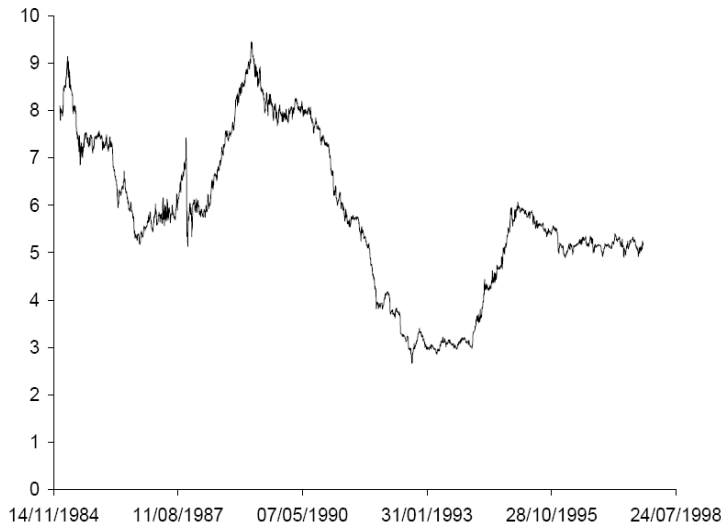
   for which we wish to obtain a model of the form

$$dr = u\left(r\right)dt + \nu r^{\beta}dW.$$

   Outline a method for doing this. Your account should be no longer than two and a half sides of A4 paper and include details of: capturing the volatility structure $w\left(r\right)$; functional form of the drift $u\left(r\right)$; slope of the yield curve to calculate the market price of risk $\lambda\left(r\right)$. **YOU ARE NOT REQUIRED TO USE REAL DATA [10 Marks]**

3. Consider a spot rate model given by

$$dr = (\eta - \gamma r)\,dt + \sqrt{\alpha r + \beta}dW,$$

where all parameters are constant. By looking for a solution of the form $Z(r, t) = e^{(A(t;T)-rB(t;T))}$ of the Bond Pricing Equation, show that the resulting pair of first order ordinary differential equations are

$$\frac{dA}{dt} = \eta B - \tfrac{1}{2}\beta B^2$$

$$\frac{dB}{dt} = \tfrac{1}{2}\alpha B^2 + \gamma B - 1,$$

with

$$A(T;T) = B(T;T) = 0.$$

By solving

$$\frac{dB}{dt} = \tfrac{1}{2}\alpha B^2 + \gamma B - 1,$$

show that

$$B(t;T) = \frac{2\left(e^{\Psi_1(T-t)} - 1\right)}{(\gamma + \Psi_1)\left(e^{\Psi_1(T-t)} - 1\right) + 2\Psi_1}$$

where

$$\Psi_1 = \sqrt{\gamma^2 + 2\alpha}.$$

[20 Marks]

4. The Vasicek model for the spot interest rate r_t is defined, by the process

$$dr_t = (\eta - \gamma r_t)\,dt + \sigma dW_t$$

where γ is the reversion rate and η/γ is the mean rate.

By solving this stochastic differential equation, calculate the mean and variance for r_t. **[7 Marks]**

5. Consider a swap with the following specification: The floating payment is at the 6 month rate, and is set six months before payment (swaplet) date. The swap expires in 5 years, and payments occur every six months on a principal of \$1. Zero-coupon bond prices are known for all maturities up to 10 years. What is the 'fair' level for the fixed rate side of the swap, so that initially the swap has no value (this should be given as an algebraic expression)? **[5 Marks]**