CQF Module 5 Assignment

JUNE 2017

Instructions

Where asked complete mathematical workings must be provided to obtain maximum credit. Each plot must have a brief explanation. Queries to Richard Diamond at *r.diamond@cqf.com*

Marking Scheme: Q1 30% **Q2** 25% **Q3** 45%

Structured Models. CDO Example

1. You are analyzing a company with the equity of \$3 million and its volatility σ_E is 50%. Company's debt of \$5 million notional matures in 1 year, and the risk-free rate is 2%. Build a structural model to report:

   (a) Computation of the firm's assets value of V_0 and volatility σ_V.
   Note: set up a system of equations in Excel/Mathematica and use Solver/alike for numerical root-finding.

   (b) Plot of 1Y PD with regard the equity volatility input σ_E. Set $K = \$5$ mil.

   Provide the plot for both, Merton PD and Black-Cox PD, both computed analytically. Briefly explain the difference *wrt* volatility levels above 60%.

2. A synthetic CDO is comprised as follows, with a capital structure of tranches:

Assets:	125 single-name CDS
Principal:	0.8 million for each name
Maturity:	5 years
1 year PD:	3% for each name
Recovery:	40% for each name

Tranche	Attachment point
Senior	7%-100%
Mezzanine	3%-7%
Equity	0%-3%

   (a) How many defaults must happen before the mezzanine tranche would experience a capital loss?

   (b) We can infer fully analytically the probability of default using the so called **one-factor model**, where Z is a common factor and w is a factor loading:

$$A_i = wZ + \sqrt{1 - w^2}\,\varepsilon_i$$

   Derive the conditional probability of default by re-arranging around the idiosyncratic variable $\varepsilon_i \sim N(0,1)$, **hint:** $\Pr(A_i \le d_i)$ is equal to the unconditional probability of default.

$$F(t = 1|Z) = \Phi\left(\frac{-1.88 - wZ}{\sqrt{1 - w^2}}\right)$$

(c) Denote K to be the number of defaults by year one, $t = 1$. Given that only two outcomes are possible (default or not), which probability distribution would K follow and why?

Conditional on **the first percentile** of the common factor Z (that, for example, represents market crash regime) and using $w = 0.3$, calculate one-year probability of capital loss in the Mezzanine tranche.

Credit Curve

Table 1 shows two small datasets from the credit markets. The set of hazard rates is typical for a highly-rated bank in stable times. DB credit spread was heightened as of 3 July 2016.

Maturity, T	λ not cum.	DF $Z(0;T)$
1Y	0.00995	0.97
2Y	0.02087	0.94
3Y	0.02579	0.92

Maturity	CDS DB EUR
1Y	141.76
2Y	165.36
3Y	188.56
4Y	207.32
5Y	218.38

Table 1: Question 1: hazard rates (left). Question 2: market data (right)

1. Given the term structure of hazard rates and discounting factors, interpolate to quarterly increments $\Delta t = 0.25$ and price the CDS with accruals on the assumption of flat spread for all tenors. $RR = 40\%$. You will have to create PL and DL computation for each quarterly period (on a spreadsheet or as code output) and use Solver to find the spread.

2. Bootstrap implied survival probabilities for DB bank with recovery rate $RR = 40\%$ on assumption that the premium paid annually in arrears (no accruals), default payment made at the end of one year period (no need for quarterly interpolation here). Use continuous DF for the risk-free rate of 0.8%.

3. Obtain the term structure of hazard rates for DB and plot Exponential *pdf* $f(t) = \lambda e^{-\lambda t}$ as appropriate for piecewise constant lambda. Describe the instability you observe.

$$P(0,T) = \exp\left(-\sum_{t=1}^{T} \lambda_t \Delta t\right) \qquad \lambda_m = -\frac{1}{\Delta t} \log \frac{P(0,t_m)}{P(0,t_{m-1})}$$

where $P(0,T)$ is a cumulative PrSurv to the end of period T, λ_m is a hazard rate per period m.

PrSurv bootstrapping from CDS quotes must be coded as a function by you – resubmission of CDS lecture spreadsheet or non-original code will receive a sizeable deduction in marks.

Interpolation

- For discounting factors, the log-linear interpolation is required. For $\tau_i < \tau < \tau_{i+1}$

$$\ln \mathrm{DF}(0,\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} \ln \mathrm{DF}(0,\tau_{i+1}) + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} \ln \mathrm{DF}(0,\tau_i)$$

  the way to read: as $\tau \to \tau_{i+1}$ the weight for $\ln \mathrm{DF}(0,\tau_i)$ goes to zero.

- Credit spreads or equally hazard rates fitted linearly or <u>by the method of your choice</u>.

$$\mathrm{CDS}(\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} \mathrm{CDS}_{i+1} + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} \mathrm{CDS}_i$$

  the assumption of a piecewise constant variable overstates the value for a concave curve and understates for the convex one.