CVA Calculation for an Interest Rate Swap

Summary

To recognise the importance of credit and counterparty risk adjustments to the derivatives business we introduce this mandatory component which must be implemented with **each topic**. One-off spreadsheet CVA computation is acceptable, but better quality work will have MtM output from Monte-Carlo.

Calculate the credit valuation adjustment, taken by Counterparty A, for an interest rate swap instrument using credit spreads for Counterparty B. Plot MtM values and produce **(a)** Expected Exposure profile. While EE is defined as $\max(\text{MtM}_\tau, 0)^+$, simulated curves allow presenting exposure distribution at each tenor and computing Potential Future Exposure at (b) the median of positive exposure and (c) 97.5th percentile. Use the output of HJM/LMM models, take a ready implementation preferably calibrated to the recent data.

Provide a brief discussion of your observations, e.g., exposure over time, location of maximum exposure, impact of very small or negative rates. The advanced sensitivity analysis will illustrate the concept of the wrong-way risk.

Step-By-Step

The inputs for IRS valuation are Forward LIBORs and discounting factors. CVA requires default probabilities: bootstrap or make reasonable assumptions to supplement the data (e.g., flat credit spreads to 5Y tenor).

- Probability of default is bootstrapped from credit spreads for a reference name (any reasonable set values) in $6M$ increment. Linear interpolation over spreads and use of ready PD bootstrapping spreadsheet are acceptable, $RR = 40\%$. CVA LGD – own choice.

- Assume the swap is written on 6M LIBOR over 5Y. Notional $N = 1$ and $\tau = 0.5$.

  To simulate the future values of L_{6M} at times $T_1, T_2, T_3, \ldots$ take either (a) HJM MC spreadsheet, calibration using 2 years of recent data preferred, (b) full calibrated LMM, or (c) ready/own calibrated one-factor model for $r(t)$. Naive Vasicek with constant parameters not recommended, use Hull & White or CIR++ which has the elasticity of variance.

- Define MtM position as Floating Leg − Fixed Leg = $(L_{6M} - K)$ appropriately discounted. Depending on your inputs, choose fixed leg (rate) K to have a positive exposure.

- Discounting factors for a static case to be taken from the OIS curve. Alternative is to use LOIS spread on simulated curves (see below).

- *CQF Lecture Valuation Adjustment - Implementation* has a simple spreadsheet with three plots: yield curve, MtM, and Expected Exposure computation for an IRS. It can be used as a starting point for your CVA calculation and analysis of multiple simulated exposures.
- *CQF Lecture Valuation Adjustment - Theory* lecture relies on xVA exercises from the textbook by Jon Gregory, as on Portal. Spreadsheet10.1 presents Expected Exposure for a swaption instrument; Spreadsheet10.2 evolves Vasicek process for $r(t)$ (constant parameters case) and presents EE/PFE for an IRS.
- If performing PCA analysis to update HJM simulation to the recent rates, please refer to the simplified technical note *PCA: Application to Yield Curves* by Richard Diamond from *CQF Lecture HJM Model*.

Forward LIBOR. OIS Discounting in Models

While based on historic volatility, the HJM offers a simple SDE, Gaussian simulation of interest rates and affine calibration with the PCA. HJM re-calibration has to be on 2-3 years of recent data of the inst. forward rates from the BLC curve (Bank of England data).

HJM and LMM output simulated forward curves. You might wish to convert from inst. forward to annualised LIBOR but that is not essential. LMM output is iterative, term structure in columns, the last column only has one terminal rate $L(T_{n-1})$, and the LIBOR curve result is taken from the diagonal. LMM SDE is example of terminal measure $\mathbb{Q}(T_n)$, whereas HJM operates under simple risk-neutral measure 'today' $\mathbb{Q}$.

Ideally discounting factors are taken from the same models (simulated curves) to match the measure. Assume constant LOIS spread and simply subtract from each simulated LIBOR curve to obtain the matching discounting curve. In practice, it might be necessary to operate with different LOIS spread for each tenor $0.5, 1, 1.5, \ldots$, the plot of such spread is referred to as *a tenor basis curve.* Note. OIS Discounting regime poses modelling challenges: (a) there are no caps traded on the OIS underlying to calibrate LMM model and (b) if undertaken from historical Forward OIS data, the second HJM calibration is best on differences $\Delta f_s = f_{Fwd,t} - f_{FwdOIS,t}$, representing 'the stochastic basis'.

One-Factor Models for $r(t)$

Calibration of one-factor interest rate models is covered in all common textbooks: fit time-dependent parameters, such as $\eta[T_{i-1}, T_i]$ with multiple instruments (bond prices). $r(t)$ models are good for quick risk management and demonstration, however, the preference for CQF Final Project is on full curve modelling.

$r(t)$ simulation drawbacks: inability to evolve the entire curve (even match the curve 'today'), calibration with constant volatility parameter prevents from matching any term structure of market volatility. One remedy to enable the use of $r(t)$ models for pricing purposes, is simulating with the stripped market volatility term structure: e.e., 3M caplet stripped volatility fitted to the a,b,c,d function).[1]

[1] $\sigma(t)$ is the function of $(T_{i-1} - t)$, where the tenor T_{i-1} comes in as a constant and function is computed piecewise for $t < T_{i-1}$.