CQF Exam 3

June 2018 Cohort

Instructions

Books and lecture notes may be referred to. Please submit a maximum of three files: one PDF (with all the answers to the problems and any theoretical developments necessary), one EXCEL file (with any numerical calculations and graphs, if required), and one ZIPPED file (with any Python, C, C++, etc, code, if required). NOTE THAT ALL ANSWERS MUST BE IN THE PDF FILE, INCLUDING GRAPHS. Failure to follow this format will result in a request for re-submission on the correct format and a possible deduction of marks. **Queries to Dr Alonso Pena (alonso.pena@fitchlearning.com)**. Good luck!

Q1. Consider a spot rate model given by

$$dr = (\eta - \gamma r)\, dt + \sqrt{\alpha r + \beta} dW,$$

where all parameters are constant. By looking for a solution of the form $Z(r,t) = e^{(A(t;T) - rB(t;T))}$ of the Bond Pricing Equation, show that the resulting pair of first order ordinary differential equations are

$$\begin{aligned} \frac{dA}{dt} &= \eta B - \tfrac{1}{2}\beta B^2 \\ \frac{dB}{dt} &= \tfrac{1}{2}\alpha B^2 + \gamma B - 1, \end{aligned}$$

with

$$A(T;T) = B(T;T) = 0.$$

By solving

$$\frac{dB}{dt} = \tfrac{1}{2}\alpha B^2 + \gamma B - 1,$$

show that

$$B(t;T) = \frac{2\left(e^{\Psi_1 (T-t)} - 1\right)}{(\gamma + \Psi_1)\left(e^{\Psi_1 (T-t)} - 1\right) + 2\Psi_1}$$

where

$$\Psi_1 = \sqrt{\gamma^2 + 2\alpha}.$$

[20 Points]

Q2. In the standard LIBOR market model (LMM) for discretely compounded interest rates, we assume that each of n spanning forward rates f_i evolves according to the stochastic differential equation (SDE):

$$\frac{df_i}{f_i} = \mu_i(\boldsymbol{f}, t)dt + \sigma_i(t)d\tilde{W}_i$$

(a) Define <u>precisely</u> each of the terms and variables in this SDE (b) Describe conceptually what is the effect of applying this SDE to the current term structure of interest rates. Support your description using diagrams and/or sketches. (c) when calibrating the LMM it is customary to parametrise correlation using the functional form:

$$\rho_{i,j} = exp[-\beta(t_i - t_j)]$$

Explain in your words what might be the rationale behind this choice of functional form. (d) Explain how we might parametrise volatility and give a functional form example.

[**20 Points**]

Q3. The Vasicek model for the spot interest rate r_t is defined, by the process

$$dr_t = (\eta - \gamma r_t)\, dt + \sigma dW_t$$

where γ is the reversion rate and η/γ is the mean rate. By solving this stochastic differential equation, calculate the mean and variance for r_t.[**10 Points**]

Q4. You are analyzing a company with the equity of \$3 million and its volatility σ_E is 50%. Company's debt of \$5 million notional matures in 1 year, and the risk-free rate is 2%. Build a structural model to report:

(a) Computation of the firm's assets value of V_0 and volatility σ_V.
Note: set up a system of equations in Excel/Mathematica and use Solver/alike for numerical root-finding.

(b) Plot of 1Y PD with regard the equity volatility input σ_E. Set $K = \$5$ mil.
Provide the plot for both, Merton PD and Black-Cox PD, both computed analytically. Briefly explain the difference *wrt* volatility levels above 60%.

[**20 Points**]

[30 **Points**]

Q5. Table 1 shows two small datasets from the credit markets. The set of hazard rates is typical for a highly-rated bank in stable times. DB credit spread was heightened as of 3 July 2016.

Maturity, T	λ not cum.	DF $Z(0;T)$
1Y	0.00995	0.97
2Y	0.02087	0.94
3Y	0.02579	0.92

Maturity	CDS DB EUR
1Y	141.76
2Y	165.36
3Y	188.56
4Y	207.32
5Y	218.38

Table 1: Question 1: hazard rates (left). Question 2: market data (right)

1. Given the term structure of hazard rates and discounting factors, interpolate to quarterly increments $\Delta t = 0.25$ and price the CDS with accruals on the assumption of flat spread for all tenors. $RR = 40\%$. You will have to create PL and DL computation for each quarterly period (on a spreadsheet or as code output) and use Solver to find the spread.

2. Bootstrap implied survival probabilities for DB bank with recovery rate $RR = 40\%$ on assumption that the premium paid annually in arrears (no accruals), default payment made at the end of one year period (no need for quarterly interpolation here). Use continuous DF for the risk-free rate of 0.8%.

3. Obtain the term structure of hazard rates for DB and plot Exponential *pdf* $f(t) = \lambda e^{-\lambda t}$ as appropriate for piecewise constant lambda. Describe the instability you observe.

$$P(0,T) = \exp\left(-\sum_{t=1}^{T} \lambda_t \Delta t\right) \qquad \lambda_m = -\frac{1}{\Delta t}\log\frac{P(0,t_m)}{P(0,t_{m-1})}$$

where $P(0,T)$ is a cumulative PrSurv to the end of period T, λ_m is a hazard rate per period m.

PrSurv bootstrapping from CDS quotes must be coded as a function by you – resubmission of CDS lecture spreadsheet or non-original code will receive a sizeable deduction in marks.

Interpolation

- For discounting factors, the log-linear interpolation is required. For $\tau_i < \tau < \tau_{i+1}$

$$\ln \mathrm{DF}(0,\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} \ln \mathrm{DF}(0,\tau_{i+1}) + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} \ln \mathrm{DF}(0,\tau_i)$$

  the way to read: as $\tau \to \tau_{i+1}$ the weight for $\ln \mathrm{DF}(0,\tau_i)$ goes to zero.

- Credit spreads or equally hazard rates fitted linearly or by the method of your choice.

$$\mathrm{CDS}(\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} \mathrm{CDS}_{i+1} + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} \mathrm{CDS}_i$$

  the assumption of a piecewise constant variable overstates the value for a concave curve and understates for the convex one.